

Exercise 49Calculate y' .

$$y = \cos\left(e^{\sqrt{\tan 3x}}\right)$$

SolutionCalculate y' by using the chain rule repeatedly.

$$\begin{aligned}y' &= \frac{d}{dx} \cos\left(e^{\sqrt{\tan 3x}}\right) \\&= -\sin\left(e^{\sqrt{\tan 3x}}\right) \cdot \frac{d}{dx}\left(e^{\sqrt{\tan 3x}}\right) \\&= -\sin\left(e^{\sqrt{\tan 3x}}\right) \cdot \left(e^{\sqrt{\tan 3x}}\right) \cdot \frac{d}{dx}\left(\sqrt{\tan 3x}\right) \\&= -\sin\left(e^{\sqrt{\tan 3x}}\right) \cdot \left(e^{\sqrt{\tan 3x}}\right) \cdot \frac{1}{2}(\tan 3x)^{-1/2} \cdot \frac{d}{dx}(\tan 3x) \\&= -\sin\left(e^{\sqrt{\tan 3x}}\right) \cdot \left(e^{\sqrt{\tan 3x}}\right) \cdot \frac{1}{2}(\tan 3x)^{-1/2} \cdot (\sec^2 3x) \cdot \frac{d}{dx}(3x) \\&= -\sin\left(e^{\sqrt{\tan 3x}}\right) \cdot \left(e^{\sqrt{\tan 3x}}\right) \cdot \frac{1}{2}(\tan 3x)^{-1/2} \cdot (\sec^2 3x) \cdot (3) \\&= -\frac{3e^{\sqrt{\tan 3x}} \sin\left(e^{\sqrt{\tan 3x}}\right) \sec^2 3x}{2\sqrt{\tan 3x}}\end{aligned}$$